

PG/INTEGRATED PG ENTRANCE EXAMINATION, APRIL 2023

MATHEMATICS

Time : Two Hours

Maximum : 200 Marks

*All questions carries 4 marks.**1 mark will be deducted for each wrong answer.*

1. Let G be a non-abelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G :

- (a) Is 6.
- (b) Is 12.
- (c) Is of the form $12k$ for $k \geq 2$.
- (d) Need not be finite.

2. Which one of the following series is divergent ?

(a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$.

(b) $\sum_{n=1}^{\infty} \frac{1}{n} \log.$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$.

(d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$.

3. The matrix equation

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$$

represents

- (a) A circle of radius $\sqrt{15}$.
- (b) An ellipse of semi major axis $\sqrt{5}$.
- (c) An ellipse of semi major axis 5.
- (d) A hyperbola.

Turn over

4. In a H.P., p^{th} term is q and the q^{th} term is p . Then pq^{th} term is :

(a) 0.

(b) 1.

(c) pq .

(d) $pq(p+q)$.

5. Newton Raphson scheme for solving $x^2 - 153 = 0$, is :

(a) $x_{n+1} = 0.5 \left[x_n + \frac{153}{x_n} \right]$.

(b) $x_{n+1} = 0.5 \left[x_n - \frac{153}{x_n} \right]$.

(c) $x_{n+1} = \left[x_n + \frac{153}{x_n} \right]$.

(d) $x_{n+1} = \left[x_n - \frac{153}{x_n} \right]$.

6. The Jacobian $\frac{d(x, y)}{d(u, v)}$ of the transformation

$$x = \frac{-u + v}{6}$$

$$y = \frac{u + 2v}{3} \text{ is}$$

(a) $\frac{1}{6}$.

(b) $-\frac{1}{6}$.

(c) 6.

(d) -6.

7. An urn contains 3 balls numbered 1, 2 and 3. The co-efficient of equation $px^2 + qx + c = 0$ is determined by drawing the numbered balls with replacement. What is the probability that the equation will have imaginary roots ?

(a) $\frac{4}{27}$.

(b) $\frac{23}{27}$.

(c) $\frac{16}{27}$.

(d) None of the above.

8. If $f(x) = x^a \log x$ and $f(0) = 0$, then the value of a for which Rolle's theorem can be applied to $[0, 1]$ is:
- (a) -2 . (b) -1 .
 (c) 0 . (d) $1/2$.
9. Which among the following is not possible?
- (a) $\exists A \text{ s.t } P(A) = \{\}$. (b) $\exists A \text{ s.t } P(A) = \{\phi\}$.
 (c) $\exists A \text{ s.t } P(A) \text{ is countable}$. (d) $\exists A \text{ s.t } P(A) \text{ is uncountable}$.
10. The number of group homomorphisms from the A_5 to the symmetric group S_4 is:
- (a) 1 . (b) 12 .
 (c) 20 . (d) 6 .
11. Which of the following is not a definition of Gamma function?
- (a) $\Gamma(n) = n!$. (b) $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$.
 (c) $\Gamma(n+1) = n \Gamma(n)$. (d) $\Gamma(n) = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1}$.
12. If a 3×3 matrix A has eigen values $1, -1$ and 0 then which among the following is not an eigen value $A^{100} + I$:
- (a) 2 . (b) 1 .
 (c) -1 . (d) All of above.
13. Let $T: \mathbb{R}^7 \rightarrow \mathbb{R}^7$ be a linear transformation with Nullity $(T) = 2$. Then, the minimum possible value of Rank (T^2) is:
- (a) 2 . (b) 1 .
 (c) 3 . (d) 4 .

Turn over

14. Which among the following iteration technique is having non linear rate of convergence :
- (a) Bisection. (b) Regula-falsi.
(c) Newton-Raphson method. (d) All of the above.
15. Which among the following functions is not uniformly continuous on the given domain :
- (a) $f(x) = x^2$ on $\mathbb{R}$. (b) $f(x) = \sin \frac{1}{x}$ on $(0, 1)$.
(c) $f(x) = \ln(x)$ on $(0, \infty)$. (d) All of the above.
16. Second degree Maclaurin polynomial of $f(x) = \frac{1}{\sqrt{x+1}}$ is:
- (a) $1 - \frac{1}{2}x + \frac{3}{8}x^2$. (b) $1 - \frac{1}{2}x + \frac{3}{4}x^2$.
(c) $1 - \frac{1}{2}x - \frac{3}{8}x^2$. (d) $1 + \frac{1}{2}x - \frac{3}{8}x^2$.
17. The product AB of any two real, symmetric matrices A and B is :
- a) Symmetric for all A and B .
(b) Never symmetric.
(c) Symmetric, if $AB = BA$.
(d) Anti-symmetric for all A and B .
18. Order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 2 & 1 & 8 & 7 & 6 & 4 & 5 \end{pmatrix}$ is:
- (a) 4. (b) 2.
(c) 6. (d) 5.
19. Which among the following is not a generator of Z_9 :
- (a) 1. (b) 2.
(c) 3. (d) 4.

20. In the Fourier series of the periodic function

$$f(x) = |\sin x|$$
$$= \sum_{n=0}^{\infty} [\alpha_n \cos nx + \beta_n \sin nx]$$

Which of the following co-efficients are non-zero ?

- (a) α_n for odd n . (b) α_n for even n .
(c) β_n for odd n . (d) β_n for even n .

21. In a restaurant, 17 men and 7 women are seated on 24 chairs at a round table. Find the total number of possible ways such that 17 men are always sitting next to each other.

- (a) $23!$. (b) $7! \times 17!$.
(c) $6! \times 16!$. (d) $8! \times 17!$.

22. The number of real solutions of $2 \sin(e^x) = 5^x + 5^{-x}$ in $[0, 1]$ is / are :

- (a) 0. (b) 1.
(c) 2. (d) Infinite.

23. Tangent line at $(1, 1)$ to the curve $x^2 + xy + 5 = 0$ is:

- (a) $y = 3x - 2$. (b) $y = -3x + 4$.
(c) $x = 3y - 2$. (d) $x = -3y + 4$.

24. The remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50} + 11^7$ is divided by 7 is :

- (a) 3. (b) 0.
(c) 6. (d) 5.

25. The Sieve of Eratosthenes is used for finding :
- (a) All primes below a given integer.
 - (b) All even numbers below a given integer.
 - (c) All odd numbers below a given integer.
 - (d) All composite numbers below a given integer.
26. Which of the following can only be used in disproving the statements ?
- (a) Direct proof.
 - (b) Contrapositive proofs.
 - (c) Counter Example.
 - (d) Mathematical Induction.
27. The argument of the Complex number $z = (1 + i\sqrt{3})(1 + i)(\cos \theta + i \sin \theta)$ is :
- (a) $\frac{\pi}{3} + \theta$.
 - (b) $\frac{\pi}{4} + \theta$.
 - (c) $\frac{7\pi}{12} + \theta$.
 - (d) None
28. Polar form of Complex number $\frac{1 + 7i}{(2 - i)^2}$ is :
- (a) $\sqrt{2}e^{(3i\pi)/4}$.
 - (b) $\sqrt{2}e^{-3i\pi/4}$.
 - (c) $\sqrt{2}e^{3i\pi/2}$.
 - (d) $\sqrt{2}$.
29. The value of $\left| \int_0^{3+i} (\bar{z})^2 dz \right|^2$, along the line $3y = x$, where $z = x + iy$ is :
- (a) $\frac{1000}{9}$.
 - (b) $\frac{1000}{27}$.
 - (c) $\frac{100}{9}$.
 - (d) $\frac{100}{3}$.

30. The value of $\int_C \frac{z^2 + 5z + 6}{(z-2)} dz$, where C is $|z| = 1$ is:
- (a) $2\pi i$. (b) 1.
(c) 0. (d) $40\pi i$.
31. For $f(x, y) = x^2 + xy - 2y - 2x + 1$, $(2, -2)$ is a:
- (a) Maximum. (b) Saddle point.
(c) Minimum. (d) None of these.
32. If u and v are harmonic functions then $f(z) = u + iv$ is:
- (a) Analytic function.
(b) Need not be analytic function.
(c) Analytic function only at $z = 0$.
(d) None of the above.
33. Fixed points for the transformation $w = \frac{z-1}{z+1}$ is:
- (a) $Z = \pm i$. (b) $Z = \pm 2i$.
(c) $Z = \pm 1$. (d) $Z = \pm 2$.
34. Image of the circle $|z| = 2$ under the transformation $w = 3z$ is :
- (a) $|w| = 2$. (b) $|w| = 3$.
(c) $|w| = 6$. (d) $|w| = 4$.

35. The bilinear transformation that maps the points $z = 0, 1, \infty$ into the points $w = -1, -2, -i, i$ respectively is :

(a) $w = \frac{(iz - 2)}{z + 2}$.

(b) $w = \frac{(iz - 2)}{z - 2}$.

(c) $w = \frac{(iz + 2)}{z + 2}$.

(d) $w = \frac{(-iz - 2)}{z + 2}$.

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2} x^n$ is :

(a) e^2 .

(b) $\frac{1}{\sqrt{e}}$.

(c) $\frac{1}{e}$.

(d) $\frac{1}{e^2}$.

37. Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$. A point at which the gradient of the function f is equal to zero is :

(a) $(-1, 1, -1)$.

(b) $(-1, -1, -1)$.

(c) $(-1, 1, 1)$.

(d) $(1, -1, 1)$.

38. Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then

the series $\sum_{n=1}^{\infty} t_n$:

(a) Diverges.

(b) Converges to $3S - a_1 - a_2$.

(c) Converges to $3S - a_1 - 2a_2$.

(d) Converges to $3S - 2a_1 - a_2$.

39. Let $M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det \left((8I - M)^3 \right)$ is:

(a) - 216.

(b) 116.

(c) 256.

(d) 216.

40. If vector field $\vec{v} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal then value of a is:

(a) 0.

(b) 3.

(c) 2.

(d) - 2.

41. The n^{th} derivative of $y = \sin px + \cos px$ is:

(a) $\sin(px + n\pi) + \cos(px + n\pi)$.

(b) $\sin\left(px + \frac{n\pi}{2}\right) + \cos\left(px + \frac{n\pi}{2}\right)$.

(c) $p^n \sin(px + n\pi) + p^n \cos\left(px + \frac{n\pi}{2}\right)$.

(d) $p^n \sin\left(px + \frac{n\pi}{2}\right) + p^n \cos\left(px + \frac{n\pi}{2}\right)$.

42. Residue of

$$\frac{z^3}{(z-1)^4(z-2)(z-3)} \text{ at } z=1 \text{ is:}$$

(a) $\frac{100}{6}$.

(b) $\frac{101}{6}$.

(c) $\frac{101}{16}$.

(d) $\frac{1}{116}$.

Turn over

43. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a/an :

- (a) Circle. (b) Parabola.
(c) Hyperbola. (d) Ellipse.

44. The values of $P_n(1)$ and $P_n(-1)$, where $P_n(x)$ is Legendra polynomial of degree n respectively are :

- (a) $(-1)^n, 1$. (b) $1, -1$.
(c) $-1, 1$. (d) $1, (-1)^n$.

45. Solution of the LP problem

Maximize $z = 2x + 6y$ subject to $-x + y \leq 1$, $2x + y \leq 2$ and $x \geq 0, y \geq 0$ is :

- (a) $\frac{4}{3}$. (b) $\frac{1}{3}$.
(c) $\frac{26}{3}$. (d) No feasible region.

46. The orthogonal trajectories of the family of curve $y = ax^2$ is (a and c being constants) :

- (a) $\frac{x^2}{1} + \frac{y^2}{2} = c$. (b) $\frac{x^2}{2} + \frac{y^2}{3} = c$.
(c) $\frac{x^2}{2} + \frac{y^2}{1} = c$. (d) $\frac{x^2}{3} + \frac{y^2}{2} = c$.

47. If the order of every element of a group is 2, then this group :

- (a) Is Abelian. (b) Is cyclic.
(c) Is of infinite order. (d) Is definitely non-abelian.

48. The sum of interior angles of a geodesic triangle on the surface of a sphere radius R is :

- (a) Less than π .
- (b) π .
- (c) Greater than π .
- (d) Not constant.

49. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric. Then :

- (a) Heine-Borel theorem holds for (X, d) .
- (b) Heine-Borel theorem does not hold for (X, d) .
- (c) X is not bounded.
- (d) X is compact.

50. Which of the following sets is in one-to-one correspondence with $\mathbb{N}$

(I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$.

(II) $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

(III) $\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$.

(IV) $\left\{\frac{p}{q} : p, q \in \mathbb{N}\right\}$.

- (a) (I) and (II).
- (b) (I), (II) and (III)
- (c) (I) and (IV).
- (d) All of the above.